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Discrete Optimization

Goal programming model for grocery shelf space allocation

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Abstract

The highly competitive grocery retail industry has annual sales of roughly half a trillion dollars in the US. While gross margins average about 28% of sales, net profits after taxes are only 1% industry-wide, causing retailers to continually search for operational improvements that increase profitability and improve customer service. One important decision that affects both of these goals is how to allocate shelf space to different products.

This paper addresses the specific problem of how to allocate a fixed amount of shelf space to different products within a particular product category, such as pickles or jelly. A nonlinear integer goal programming formulation is proposed that considers both profitability and customer service factors. This decision support tool shows the tradeoffs between increased profitability and improved customer service, and allows the manager to make the best tradeoff for the situation. An alternate approach is also proposed.

Keywords: Goal programming; Retailing; Shelf space allocation; Grocery industry

1. Introduction

The US grocery industry is highly competitive, with annual sales in 2003 of around half a trillion dollars. According to a 2003 annual report of the US grocery industry, average gross margins of grocery retail stores was about 28% of sales, but net profit after taxes was only about 1% (Food Marketing Institute, 2004). Because of the competition and low profit margins, grocery chains are continuously searching for ways to reduce costs, increase sales, and improve customer service.

At the grocery retail store level, two performance measures that are of greatest importance are profitability and customer service. When making changes to improve profitability, management must be cautious of any resulting negative impacts on customer service. Good customer service is critical to maintaining the loyalty of customers.

One issue that has an impact on both profitability and customer service is shelf space allocation, which is part of category management. A well-managed

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shelf space not only improves customer service by reducing stockouts; it can also improve the return on inventory investment by increasing sales and profit margins (Yang and Chen, 1999; Yang, 2001).

The purpose of the shelf space allocation decision is to improve the financial performance of the store (Buttle, 1984), however this decision is typically made infrequently, maybe twice a year. For a small portion of the products in a store, such as soft drinks and breakfast cereal, shelf space is negotiated with manufacturers or distributors. However for most products, the decision is up to the grocery store or its corporate office, and is based on demand rates and profit potential.

The first part of the decision is how much shelf space to allocate to a particular product category, such as olive oil or canned chili. The next part of the decision would be how much shelf space to allocate to each different product, given a fixed amount of shelf space for that product category.

In this paper we focus on the second part of the decision, which assumes that the total shelf space for a particular category has already been determined. The problem becomes how much of that total shelf space to assign to each of a given number of different products. A trivial solution, known as the "sales productivity method" (Mason and Mayer, 1990), would be to assign the total category shelf space in proportion to the demand rate of each product (assuming there is enough total shelf space to avoid frequent stockouts). However, this approach ignores two well known consumer behavior phenomena: price sensitivity and shelf space sensitivity.

This paper proposes a shelf space allocation model that considers the tradeoff between profitability and customer service level. The formulation is a nonlinear integer weighted goal program, which allows the manager to evaluate the tradeoff between profitability and customer service level to select the most preferred solution. An experiment is presented that assesses the usefulness of this tool over different parameter values. An alternate approach is also proposed, instead of using goal programming.

2. Price and shelf space sensitivity

It is well known in the marketing field that price is a key factor when a consumer selects among similar competing products, although it is not the only factor. Other factors that impact a consumer's choice among alternatives include perceived quality of the product, brand reputation, personal loyalty to a brand, desire to try something different, and advertising, among others. Examples of consumer behavior research that consider how prices are involved in a consumer's product/brand choice include research by Dickson and Sawyer (1984); Erdem et al. (2004); Lemon and Nowlis (2002); Suri and Monroe (2003). In general, consumers would prefer to pay less for a product rather than more. But considering all the other factors that affect consumer choice, all consumers do not buy the cheapest alternative.

For example, consider four brands of green olives that are each priced at 99 cents per jar. These four products would each have different average demand rates because of the other factors mentioned above. If the price of one brand is now changed to 97 cents, then a small percentage of consumers (those who are highly price sensitive) would switch over to this brand instead of the brand they might otherwise have bought. If the price is then changed to 95 cents, then an additional percentage of consumers would switch. This simple example illustrates the basic concept of price sensitivity: lowering the price of one alternative will increase the demand for that alternative.

No single functional form for the relationship between demand and price fits all situations. In this research we represent the impact of price sensitivity of demand through a price sensitivity factor (Π), which compares the price of a particular product with the average price of all products in the category. We divide the average category price (p_{avg}) by the price of a particular product (p_i), and then modify this ratio by a specified exponent (γ).

$$\text{Price sensitivity factor}: \Pi = \left(\frac{p_{\text{avg}}}{p_i}\right)^{\gamma}. \tag{1}$$

A value of 0.5 for γ was used for the examples in this study, but any exponent value could be used, as appropriate for the situation. A product with no price premium or discount ($p_i = p_{\text{avg}}$) would have a price sensitivity factor of 1.0, which is a desirable characteristic of this form.

The second consumer behavior phenomenon considered is the impact on demand due to shelf space sensitivity. This refers to the observed behavior of some consumers to buy the product alternative that has the highest displayed inventory level. This approach is used in many experimental studies (e.g. Yang and Chen, 1999) and is defined by Curhan (1973) as the ratio of the relative change in unit sales to the relative change in shelf space. One explanation for this behavior is that when some consumers see one product having more displayed inventory on the shelf than the competing products, they think that this product must be more popular and should provide less risk of disappointment, so they decide to buy this product. The theory in self-service grocery retailing is that demand for a product is influenced by the amount of display exposure, and it has been speculated that this structure of promotion is capable of shifting brand choices among consumers (Anderson, 1979; Urban, 2002).

Corstjens and Doyle (1981) and Corstjens and Doyle (1983) referred to this type of behavior as individual space elasticity. They represent this demand component as s^{β}, where s is the amount of shelf space and the exponent β is a constant elasticity parameter (values around 0.1 were used for β in their example). With this component, increasing the shelf space for a product will slightly increase its demand rate. Corstjens and Doyle multiplied this space elasticity component by other components to obtain a resulting demand rate. They note that general polynomial forms for demand are common in previous theoretical work and supported by broad empirical findings.

In the current study we propose a slightly different polynomial form for shelf space sensitivity. First, a base level shelf space allocation is determined for product i by taking the base-level demand for product i (d_i) and dividing this by the demand for all products in the category (d_{total}), and then multiplying this fraction by the total category shelf space available (S_N).

$$\text{Base level allocation: } S_i^0 = \left(\frac{d_i}{d_{\text{total}}}\right) S_N. \tag{2}$$

This base level allocation is similar to what Larson and DeMarais (1990) called "psychic stock," which is displayed inventory carried to stimulate demand. Next, a shelf space sensitivity factor (Σ) is composed by dividing the actual shelf space allocation for a product (S_i) by the base level allocation (S_i^0), and modifying this ratio by an exponent parameter (λ) to estimate the shelf space sensitivity.

$$\text{Shelf space sensitivity factor: } \Sigma = \left(\frac{S_i}{S_i^0}\right)^{\lambda}. \tag{3}$$

Based on our literature review of shelf space sensitivity, similar exponent parameter values from past research have been 0.212 (Curhan, 1973), 0.15 (Hansen and Heinsbrooek, 1979), 0.086 (Corstjens and Doyle, 1981), and 0.8 (Desmet and Renaudin, 1998). A value of 0.5 for λ was used for the examples in this study, but any exponent value could be used, as appropriate for the situation. A desirable characteristic of this form is that a product having the base level allocation ($S_i = S_i^0$) would have a shelf space sensitivity factor of 1.0.

A resulting demand function is developed by multiplying a base level demand rate by the price and shelf space sensitivity factors. The base level demand rate for a product (d_i) is the demand rate expected if all prices of the category products are the same and the base level shelf space allocation is used. The resulting demand rate (D_i) for product i is computed as follows:

$$D_i = d_i \Pi \Sigma = d_i \left(\frac{p_{\text{avg}}}{p_i}\right)^{\gamma} \left(\frac{S_i}{S_i^0}\right)^{\lambda}. \tag{4}$$

3. Goal programming model development

A goal programming model for the shelf space allocation problem is developed that weights two objectives related to profitability and customer service. The profitability measure used is gross profit margin. The type of customer service measure used relates to having an amount of shelf space for each product in proportion to the relative demand rate for that product compared with the other category products. The rationale behind this type of measure is that store managers would prefer to always have the full variety of their offerings in stock, and if the shelf space allocation is too low then the chances of stocking out increase. In this sense it is somewhat undesirable for allocations to be lower than the base level shelf space allocations.

To summarize the two objectives used, maximizing profitability will attempt to allocate more shelf space to those products with higher profit margins and less space to those with lower margins, while minimizing under-base-level allocations will attempt to allocate the base level shelf space allocation to each product. A tradeoff clearly exists between the profitability and customer service level objectives. Goal programming is useful to analyze this type of tradeoff and allow the store manager to determine the preferred tradeoff level.

Before presenting the model, the parameters and variables used are described below for easy reference.

Parameters
c_i = purchase cost per unit of product i,
d_i = base level demand rate for product i (units per day),
n = number of products in the category,
p_i = selling price per unit for product i,
p_{avg} = average selling price for category (sum of p_i for all products, divided by n),
S_N = total shelf space available for category (number of product units),
S_i^0 = base level shelf space allocation for *product* i (number of product units),
w = weight given to profit maximization objective (values in 0–1 range),
$(1 - w)$ = weight given to under-base-level allocation objective,
μ = minimum fraction of base level allocation allowed (values in 0–1 range).

Variables
S_i = actual shelf space allocation for product i (number of product units),
D_i = resulting demand rate for product i (units per day),
u_i = underachievement goal variable for product i allocation,
o_i = overachievement goal variable for product i allocation.

The goal objective function maximizes the profit components and minimizes the base-level allocation underachievement components (hence the minus sign in front of the underachievement components).

Maximize:

$$w[\text{total profits}] - (1-w) \times [\text{total underachievement of base-level allocations}], \tag{5}$$

where

$$\begin{aligned}[\text{total profits}] &= (\text{gross margins}) \times (\text{resulting demand rates}) \\ &= \sum_{i=1}^{n}\{(p_i - c_i)D_i\}, \\ &= \sum_{i=1}^{n}\{(p_i - c_i)d_i \Pi\Sigma\}, \\ &= \sum_{i=1}^{n}\left\{(p_i - c_i)d_i\left(\frac{p_{\text{avg}}}{p_i}\right)^{\gamma}\left(\frac{S_i}{S_i^0}\right)^{\lambda}\right\}.\end{aligned} \tag{6}$$

The goal constraints specify only one goal, which is the base level allocations: S_i^0. The goal programming formulation is shown in Eq. (7) through Eq. (11).

$$\text{Maximize:} \quad w\sum_{i=1}^{n}\left\{(p_i - c_i)d_i\left(\frac{p_{\text{avg}}}{p_i}\right)^{\gamma}\left(\frac{S_i}{S_i^0}\right)^{\lambda}\right\} - (1-w)\sum_{i=1}^{n} u_i, \tag{7}$$

Subject to:

$$\sum_{i=1}^{n} S_i = S_N, \tag{8}$$

$$S_i + u_i - o_i = S_i^0, \quad \text{for each } i, \tag{9}$$

$$S_i \geqslant \mu S_i^0, \quad \text{for each } i, \tag{10}$$

$$S_i,\ u_i,\ o_i \geqslant 0; \quad S_i \text{ variables are integers.} \tag{11}$$

Only the objective function (7) contains nonlinear terms (i.e., S_i^{λ}). Constraint (8) requires the total available category shelf space (S_N) to be allocated. The constraints in formula (9) are the goal constraints for the base level allocations. The underachievement of base level allocation for product i is represented by u_i, and the overachievement by o_i. Since no nonlinear terms are used in the constraints, the feasible solution space is approximately linearly bounded but with integrality conditions. Also, the nonlinear terms S_i^{λ} in the objective function strictly increase as S_i increase. These characteristics should lead to the obtained solution being a global optimal.

The constraints in formula (10) are included to prevent an unreasonably low shelf space allocation for any product. Any value could be used for μ, as deemed appropriate by management, to impose a minimum level of shelf space for each product. A value of $\mu = 0.5$ was used in this study, so that at least 50% of the base level allocation for each product was required. Without such a restriction, some low profit margin products could end up with a zero allocation of shelf space. A value of $\mu = 0.0$ was also examined for comparison.

The size of the formulation is quite small. For n products in a category, the number of real variables (o_i, u_i) is $2n$, the number of integer variables (S_i) is n, and the number of constraints is $2n + 1$.

4. Alternate approach

To avoid the decision maker having to decide on the desired weight to use with the goal programming model, an alternate approach is proposed that attempts to achieve the best compromise between the two objectives: maximizing profits and minimizing MAD. MAD is the mean absolute deviation between actual shelf space allocations and base level allocations, as described later. The following variables are used to describe this approach:

P_j = total category profit for any solution j,
P_b = best (optimal) total profit (achieved when MAD = M_w),
P_w = worst total profit (achieved when MAD = M_b, at base level allocations),
$P\%_j$ = percent of optimal profit achieved with solution j,
M_j = mean absolute deviation (MAD) for solution j,
M_b = best (optimal) value of MAD (achieved when profit = P_w) (M_b always equals zero, at base level allocation),
M_w = worst value of MAD (achieved when profit = P_b),
$M\%_j$ = percent of optimal MAD value achieved with solution j,
AOA_1 = average optimal achievement [$(P\%_j + M\%_j)/2$],
AOA_2 = average optimal achievement (using MSE rather than MAD).

The alternate model is a nonlinear integer program, but not a goal program. The objective function is to maximize AOA_1 (or AOA_2). AOA achieves the maximum average percent achievement of the two optimal solutions for profit and MAD, as compared with the worst solution for each. [*Note*: This criterion of optimizing combined percent achievements for the two objectives is different than setting $w = 0.5$ in formula (5).] Higher values of profit are preferred, and lower values of MAD are preferred. P_w and M_b are obtained using the base level allocation solution (S_i^0). P_b and M_w are obtained by solving the following model for optimal profit:

$$\text{Maximize profit} = P_j = \sum_{i=1}^{n}\left\{(p_i - c_i)d_i\left(\frac{p_{avg}}{p_i}\right)^{\gamma}\left(\frac{S_i}{S_i^0}\right)^{\lambda}\right\}, \tag{12}$$

Subject to:

$$\sum_{i=1}^{n} S_i = S_N, \tag{13}$$

$$S_i \geqslant \mu S_i^0, \quad \text{for each } i, \tag{14}$$

$$S_i \geqslant 0; \quad S_i \text{ variables are integers.} \tag{15}$$

The following formulation is then used to maximize AOA_1 (or AOA_2):

$$\text{Maximize } AOA_1 = 100 * \frac{P\%_j + M\%_j}{2}, \tag{16}$$

Subject to:

$$\sum_{i=1}^{n} S_i = S_N, \tag{17}$$

$$S_i \geqslant \mu S_i^0, \quad \text{for each } i, \tag{18}$$

$$P_j = \sum_{i=1}^{n}\left\{(p_i - c_i)d_i\left(\frac{p_{avg}}{p_i}\right)^{\gamma}\left(\frac{S_i}{S_i^0}\right)^{\lambda}\right\}, \tag{19}$$

$$P\%_j = \frac{P_j - P_w}{P_b - P_w}, \tag{20}$$

$$M_j = \frac{1}{n}\sum_{i=1}^{n} |S_i - S_i^0|, \tag{21}$$

$$M\%_j = \frac{M_w - M_j}{M_w - M_b}, \tag{22}$$

$$\text{All variables} \geqslant 0,$$

$$S_i \text{ variables are integers.} \tag{23}$$

The absolute value operator in formula (21) can be removed by replacing $S_i - S_i^0$ with $R_i^+ + R_i^-$ and adding the constraint: $S_i - S_i^0 = R_i^+ - R_i^-$, where $R_i^+, R_i^- \geqslant 0$.

In solving for AOA_2, the only change is that formula (21) is replaced with the following constraint:

$$M_j = \frac{1}{n}\sum_{i=1}^{n}(S_i - S_i^0)^2. \tag{24}$$

5. Experimental design

To assess the effectiveness of the proposed model and to determine under what circumstances it is most useful, we designed a full factorial experiment to evaluate the impact of different parameter values. In order to model a realistic environmental setting, this experimental design is based on the data collection of a field study conducted over a three-month period at a US grocery store in the Dallas, Texas area (Reyes, 2002).

The experiment is limited to one product category. The number of products in the category is set at 4. The purchase costs from the warehouse are \$1.35, 1.45, 1.55, and 1.65, for products 1, 2, 3 and 4, respectively. The total available category shelf space is set to hold 120 units (which is equivalent to 5 days worth of demand on average).

Three levels of profit margin percentages are evaluated. For the first level, all products have a 35% gross profit margin. For the second level, the four profit margins are 20%, 30%, 40% and 50%, for products 1, 2, 3 and 4, respectively. The four profit margins are alternatively evaluated as 50%, 40%, 30% and 20%, for products 1, 2, 3 and 4, respectively.

Three levels are used for demand rates. For the first level, all product demand rates are 6 units per day. For the second level, the rates are 3, 5, 7, and 9 units per day, for products 1, 2, 3 and 4, respectively. The four demand rates are alternatively evaluated as 9, 7, 5 and 3 units per day, for products 1, 2, 3 and 4, respectively.

Values for profit margin percentages and demand rates are sequenced from either lowest to highest or from highest to lowest (when that parameter had varying values) to help identify circumstances in which the objective tradeoffs are more important. These experimental conditions along with the parameter levels described above result in four problem sets with a total of nine problems, as illustrated in Table 1. Problem set 1 includes case 1a2a, where demand rate and profit margin percentages are the same for all products. Problem set 2 includes case 1a2b and case 1a2c, where only demand rates are different for products. Problem set 3 includes case 1b2a and case 1c2a, where only profit margin percentages are different for products. Finally, Problem set 4 includes case 1b2b, case 1b2c, case 1c2b, and case 1c2c, where both demand rates and profit margin percentages vary.

The three different levels each for mix of profit margin percentages and mix of demand rates results in $3 \times 3 = 9$ experimental scenarios. With each scenario, the weight parameter in the goal programming objective function was varied over several levels to develop a range of different solutions to show the tradeoffs between the two objectives of profitability and underachievement of base level allocations. A weight of 0.0 optimizes the underachievement goal (but gives the worst profit), and a weight of 1.0 optimizes profitability (but gives the worst underachievement of base level allocations).

To obtain a single measure that reflects the combined deviation from base level allocations for all products, we use mean absolute deviation (MAD). MAD is the average over all products of the absolute difference between a product's allocated amount of shelf space and that product's base level of shelf space. MAD is expressed in product units, and is directly proportional to the average level of underachievement of base level allocations (MAD/2). So for the base level allocations for all products, the MAD value would be zero, which is the optimal underachievement goal. Mean squared deviation (MSE) is also considered with the alternate approach.

6. Results and analysis

For each of the nine experimental problems we gradually vary the weight parameter in the objective function so that the solutions gradually change from completely satisfying the underachievement goal to completely satisfying the profitability goal, to identify the tradeoffs that are made between objectives. The following tables show the amount of shelf space

Table 1
Experimental design

		Product demand rates (d_i)		
		Constant 6, 6, 6, 6 (2a)	Increasing 3, 5, 7, 9 (2b)	Decreasing 9, 7, 5, 3 (2c)
Profit margins	Constant 35%, 35%, 35%, 35% (1a)	Case 1a2a (Set 1)	Case 1a2b (Set 2)	Case 1a2c (Set 2)
	Increasing 20%, 30%, 40%, 50% (1b)	Case 1b2a (Set 3)	Case 1b2b (Set 4)	Case 1b2c (Set 4)
	Decreasing 50%, 40%, 30%, 20% (1c)	Case 1c2a (Set 3)	Case 1c2b (Set 4)	Case 1c2c (Set 4)

Note: Margins and demand rates are listed for products 1, 2, 3, and 4 respectively.

(S_i is shown as number of product units on shelf) allocated to each product for each solution with different weights, as well as the total profit and MAD value. Also shown are columns with solutions for AOA_1, for AOA_2, and for no lower limit on shelf space allocations with weight = 1.0 (Unrestr).

6.1. Problem set 1

In Problem set 1 (case 1a2a) we keep the demand rates and the profit margin percentages constant. The results are shown in Table 2. In this scenario, slightly more shelf space is allocated to the items with higher purchase costs (products 3 and 4) because they have higher resulting profit contribution margins (even though the percentage is fixed). While the base allocations are equal for all products, as we increase the weight for category profits we find a small increase in mean absolute deviation (MAD) from the base allocation and very slight increase in category profits. Since demand rates and profit margin percentages are equal for all products, these slight differences are due to the different unit cost of each product: \$1.35, 1.45, 1.55, and 1.65, for products 1, 2, 3 and 4, respectively (constant for all product sets).

6.2. Problem set 2

In Problem set 2 (1a2b and 1a2c) we vary the demand rates, while keeping the profit margin percentages constant. The results are presented in Table 3, for both increasing and decreasing demand rate cases. The base allocations increase proportionally to the increasing demand rates. As we increase the weight on category profits, we find slightly higher allocations for products with higher purchase costs and higher demand rates (product 3 and/or 4). As with the Problem set 1, changing the weight from 0 to 1 resulted in very little improvement in category

Table 2
Constant demand rates and constant profit margin percentages

Base		Weight						Unrestr	AOA_1	AOA_2
		0.000	0.996	0.997	0.998	0.999	1.000			
Case 1a2a: Demand = 6, 6, 6, 6; margin% = 35, 35, 35, 35										
30	S_1	30	29	29	28	27	27	27	28	28
30	S_2	30	30	30	30	30	29	29	30	30
30	S_3	30	30	30	30	31	31	31	30	31
30	S_4	30	31	31	32	32	33	33	32	31
	Profit:	12.591	12.596	12.596	12.598	12.599	12.600	12.600	12.598	12.597
	MAD:	0.00	0.50	0.50	1.00	1.50	2.00	2.00	1.00	1.00

Table 3
Varying demand rates and constant profit margins percentages

Base		Weight						Unrestr	AOA_1	AOA_2
		0.000	0.996	0.997	0.998	0.999	1.000			
Case 1a2b: Demand = 3, 5, 7, 9; margin% = 35, 35, 35, 35										
15	S_1	15	14	14	14	14	13	13	14	14
25	S_2	25	25	25	24	24	24	24	25	25
35	S_3	35	35	35	35	35	35	35	35	35
45	S_4	45	46	46	47	47	48	48	46	46
	Profit:	12.766	12.771	12.771	12.773	12.773	12.774	12.774	12.771	12.771
	MAD:	0.00	0.50	0.50	1.00	1.00	1.50	1.50	0.50	0.50
Case 1a2c: Demand = 9, 7, 5, 3; margin% = 35, 35, 35, 35										
45	S_1	45	44	44	43	42	42	42	44	43
35	S_2	35	35	35	35	35	35	35	35	35
25	S_3	25	25	25	26	26	26	26	25	26
15	S_4	15	16	16	16	17	17	17	16	16
	Profit:	12.416	12.420	12.420	12.422	12.424	12.424	12.424	12.420	12.422
	MAD:	0.00	0.50	0.50	1.00	1.50	1.50	1.50	0.50	1.00

profits with only a moderate increase in MAD. This result was similar for both increasing and decreasing demand rates.

6.3. Problem set 3

In Problem set 3 (1b2a and 1c2a), we vary the profit margin percentages while keeping the demand rates constant. Table 4 shows the results for both increasing and decreasing profit margin percentages. In the former case (1b2a), increasing the weight from 0 to 1 increases the profit by 4.5% (from 12.626 to 13.195), as MAD increases from 0 to 13. In the latter case (1c2a), increasing the weight from 0 to 1 increases the profit by 2.9% (from 12.287 to 12.649), as MAD increases from 0 to 11. In both cases, products with greater profit margin percentages receive higher shelf space allocations as the weight on profit increases. Figs. 1 and 2 show this tradeoff graphically for cases 1b2a and 1c2a.

The column labeled Unrestr is for the solution with a weight of 1.0 but no lower bound on the minimum shelf space allocation. All other columns use a lower bound of 50% of the base level allocation. Relaxing this bound in case 1b2a results in a profit increase from 13.195 to 13.241, with the lowest product allocation at 30% of base level (Product 1: 9/30).

Column AOA_1 shows the solution for the alternate approach that maximizes the average percentage achievement for optimal profit and MAD. In case 1b2a the AOA_1 solution represents 83.97%

Table 4
Constant demand rates and varying profit margin percentages

Base		Weight						Unrestr	AOA_1	AOA_2
		0.000	0.960	0.970	0.980	0.990	1.000			
Case 1b2a: Demand = 6, 6, 6, 6; margin% = 20, 30, 40, 50										
30	S_1	30	28	22	16	15	15	9	15	19
30	S_2	30	30	30	30	26	19	20	30	27
30	S_3	30	30	30	30	31	34	36	30	34
30	S_4	30	32	38	44	48	52	55	45	40
	Profit:	12.626	12.712	12.929	13.084	13.194	13.195	13.241	13.103	13.044
	MAD:	0.00	1.00	4.00	7.00	12.50	13.00	15.50	7.50	7.00
Case 1c2a: Demand = 6, 6, 6, 6; margin% = 50, 40, 30, 20										
30	S_1	30	30	32	38	43	47	49	42	38
30	S_2	30	30	30	30	31	35	36	30	33
30	S_3	30	30	30	30	30	23	23	30	28
30	S_4	30	30	28	22	16	15	12	18	21
	Profit:	12.287	12.287	12.352	12.507	12.602	12.649	12.660	12.577	12.542
	MAD:	0.00	0.00	1.00	4.00	7.00	11.00	12.50	6.00	5.50

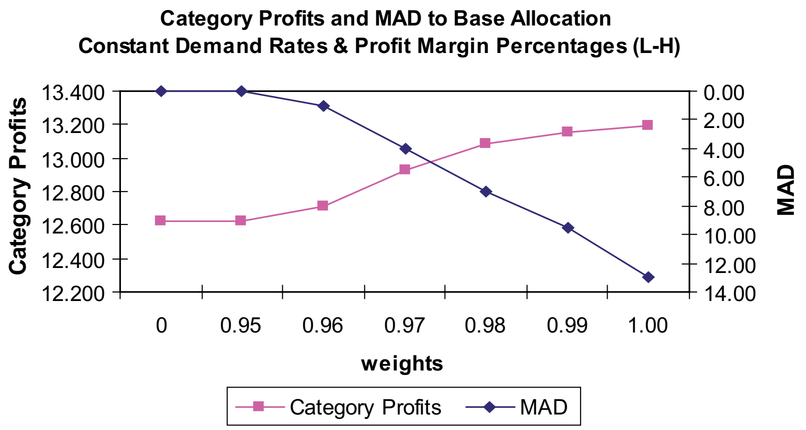

Fig. 1. Category profits and MAD to base allocation for case 1b2a.

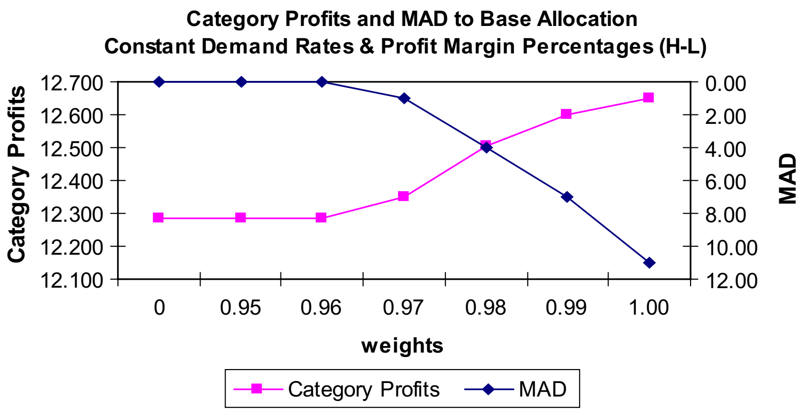

Fig. 2. Category profits and MAD to base allocation for case 1c2a.

optimal profit achievement and 42.31% optimal MAD achievement, yielding 63.14% average optimal achievement (AOA). In case 1c2a the AOA_1 solution represents 80.01% optimal profit achievement and 45.45% optimal MAD achievement, yielding 62.73% AOA. As a comparison, the solution in the weight = 0.0 column represents 0.0% optimal profit achievement and 100% optimal MAD achievement, yielding 50% AOA. And the solution in the weight = 1.0 column represents 100% optimal profit achievement and 0.0% optimal MAD achievement, also yielding 50% AOA.

Column AOA_2 shows the solution for the alternate approach using MSE instead of MAD. In case 1b2a the AOA_2 solution represents 73.43% optimal profit achievement and 70.92% optimal MSE achievement, yielding 72.18% AOA. In case 1c2a the AOA_2 solution represents 70.42% optimal profit achievement and 73.13% optimal MSE achievement, yielding 71.78% AOA.

6.4. Problem set 4

Finally, in Problem set 4 (1b2b, 1b2c, 1c2b, and 1c2c) we vary both the demand rates and the profit margin percentages. Table 5 shows the results for the four combinations of demand rate sequence and profit margin percentage sequence. As the weight varies from 0 to 1, improvements in profit range from 1.9% (case 1c2c) to 5.2% (case 1b2c). The greatest increase in profit was for the case where products with higher demand rates and lower purchase costs had lower profit margin percentages (case 1b2c). With a weight of 1.0, the MAD values ranged from 8.0 for case 1c2c to 14.5 for case 1c2b. As with Problem set 3, more shelf space is allocated to products with higher profit margins percentages as the weight on profit increases. Table 6 gives the percentage achievements for profit, MAD, and MSE, as well as the average achievements for each case, for AOA_1 and AOA_2.

7. Summary and conclusions

The shelf space allocation problem was analyzed in this study as it affects retail store performance. Two conflicting objectives were evaluated, total category profits and a measure of customer service level. An experiment was designed to evaluate these objectives over different ranges of parameter values. Table 7 shows a summary of experimental results.

When profit margin percentages are the same for all products, deviations from base-level shelf space allocations (i.e., base level allocations assign shelf space in exact proportions to demand rates) result in insignificant improvement in category profits. So under the condition of equal profit margin percentages for all products, demand should be set at the base level allocations.

On the other hand, when profit margin percentages are not equal for all products, then a significant increase in category profits can be gained by deviating from base level allocations and allocating more shelf space to products with higher profit margin percentages. With the proposed goal programming approach, it would be up to the category manager to determine the best level of tradeoff. With the proposed alternate approach the best allocations are made based on the percent achievements of optimality for the two objectives. The goal programming approach provides flexibility in selecting the best level of objective tradeoffs, while the alternate

Table 5
Varying demand rates and varying profit margin percentages

Base		Weight						Unrestr	AOA_1	AOA_2
		0.000	0.960	0.970	0.980	0.990	1.000			
Case 1b2b: Demand = 3, 5, 7, 9; margin% = 20, 30, 40, 50										
15	S_1	15	13	9	8	8	8	4	8	8
25	S_2	25	25	25	23	17	13	14	22	21
35	S_3	35	35	35	35	35	33	34	35	37
45	S_4	45	47	51	54	60	66	68	55	54
	Profit:	14.112	14.197	14.342	14.418	14.513	14.536	14.588	14.438	14.440
	MAD:	0.00	1.00	3.00	4.50	7.50	10.50	11.50	5.00	5.50
Case 1b2c: Demand = 9, 7, 5, 3; margin% = 20, 30, 40, 50										
45	S_1	45	44	39	31	23	23	17	31	33
35	S_2	35	35	35	35	35	28	30	35	33
25	S_3	25	25	25	28	32	36	38	28	30
15	S_4	15	16	21	26	30	33	35	26	24
	Profit:	11.140	11.184	11.364	11.563	11.688	11.715	11.740	11.563	11.539
	MAD:	0.00	0.50	3.00	7.00	11.00	14.50	16.50	7.00	7.00
Case 1c2b: Demand = 3, 5, 7, 9; margin% = 50, 40, 30, 20										
15	S_1	15	15	16	21	25	29	29	23	22
25	S_2	25	25	25	26	31	36	36	29	30
35	S_3	35	35	35	35	35	32	32	35	34
45	S_4	45	45	44	38	29	23	23	33	34
	Profit:	11.153	11.153	11.186	11.338	11.478	11.523	11.524	11.427	11.417
	MAD:	0.00	0.00	0.50	3.50	8.00	12.50	12.50	6.00	6.00
Case 1c2c: Demand = 9, 7, 5, 3; margin% = 50, 40, 30, 20										
45	S_1	45	45	47	51	56	61	62	52	52
35	S_2	35	35	35	35	35	35	36	35	37
25	S_3	25	25	25	25	21	16	17	24	22
15	S_4	15	15	13	9	8	8	5	9	9
	Profit:	13.420	13.420	13.485	13.586	13.657	13.681	13.701	13.602	13.619
	MAD:	0.00	0.00	1.00	3.00	5.50	8.00	9.00	3.50	4.50

Table 6
Percentage achievements for alternate approaches AOA_1 and AOA_2

	Case 1b2b		Case 1b2c		Case 1c2b		Case 1c2c	
	AOA_1	AOA_2	AOA_1	AOA_2	AOA_1	AOA_2	AOA_1	AOA_2
Profit%	76.83	77.23	73.51	69.27	73.96	71.25	69.76	76.26
MAD%	52.38		51.72		52.00		56.25	
MSE%		76.49		74.03		75.80		74.61
AOA	64.60	76.86	62.62	71.65	62.98	73.53	63.00	75.43

approach serves as an efficient heuristic to obtain a good compromise solution.

Future research opportunities include investigating alternate ways of combining profitability and customer service objectives into a single metric to optimize. Other factors that may affect the tradeoffs between profitability and customer service should also be investigated, such as variability of demand, probability of stockouts, and selection of product mix. Since only one measure of customer service was used here, it may be useful to investigate other measures of customer service as well. Another research opportunity is to extend the proposed methodology to address the space allocation problem among multiple categories of products simultaneously. This would lead to better space utilization

Table 7
Summary of experimental results

(Prob. set) Problem	Factors varied	Profits (wt = 0.0)	Profits AOA_1	Profits (wt = 1.0)	Change in profits	MAD AOA_1	MAD (wt = 1.0)
(1)							
1a2a	Marg –, Dem –	12.591	12.598	12.600	0.009	1.0	2.0
(2)							
1a2b	Marg –, Dem ↑	12.766	12.771	12.774	0.008	0.5	1.5
1a2c	Marg –, Dem ↓	12.416	12.420	12.424	0.008	0.5	1.5
(3)							
1b2a	Marg ↑, Dem –	12.626	13.103	13.195	0.569	7.5	13.0
1c2a	Marg ↓, Dem –	12.287	12.577	12.649	0.362	6.0	11.0
(4)							
1b2b	Marg ↑, Dem ↑	14.112	14.438	14.536	0.424	5.0	10.5
1b2c	Marg ↑, Dem ↓	11.140	11.563	11.715	0.575	7.0	14.5
1c2b	Marg ↓, Dem ↑	11.153	11.427	11.523	0.370	6.0	12.5
1c2c	Marg ↓, Dem ↓	13.420	13.602	13.681	0.261	3.5	8.0

for the entire store rather than just for one product category.

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