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After-sales service competition in a supply chain: Optimization of customer satisfaction level or profit or both?

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ABSTRACT

For durable consumer products, after-sales services play an important role in customers' purchase decisions. A manufacturer offers basic warranty available to all customers who buy the product, while a retailer offers optional after-sales service that is available only to customers who pay for the option. We explore the interaction of these two after-sales services assuming two customer segments. Formulating five analytical models, we found that after-sales service plans that are determined to maximize profits do not match optimal after-sales service levels that can satisfy customers the most.

1. Introduction

Consider the following computer sales situation. A customer buying a personal computer (PC) at a computer shop is asked by the shop employee whether he wants to purchase certain after-sales service options, such as a longer warranty than usual. In most cases, PC manufacturers already offer a manufacturer's warranty, which is generally the default for every customer, whose support is limited, usually to a single year. For example, notebook computers sold at dell.com have a default hardware warranty that states "1 Yr Ltd Warranty, 1 Yr Mail-In Service, and 1 Yr Technical Support." At the same time, dell.com offers customers optional after-sales service plans, such as "3 Year Basic Service Plan" that includes a "$50 Dell Promo Gift Card" at the additional price of $119.00 (dell.com web site, 2010). Some customers may be sensitive to price and think the base warranty is sufficient; however, other customers may pay much more attention to after-sales service and would welcome such an additional service option with an extra payment. Hence, it is critical for both the retailer and the manufacturer to create a reasonable after-sales policy that will result in the highest level of customer satisfaction.

An optional after-sales service plan is an important business tool for durable goods, such as electric appliances and PCs. It is well known that the margin from after-sales service is much larger than that from the product. That is, after-sales service is considered a key revenue generator in certain categories (Cohen et al., 2006; Cohen and Whang, 1997). Also, after-sales service is now considered a critical strategic tool in the automobile industry (Flees and Senturia, 2008). Hence, on one hand, offering a large number of extra after-sales service plans to consumers leads to higher profitability. Recent marketing management focuses on lifetime value of a customer and maintaining long-term relationships with customers (Gupta and Lehmann, 2007). From this customer-relationship viewpoint, after-sales service is regarded as an important factor that has an impact on establishing good relationships with customers. On the other hand, a default and free basic after-sales service, such as a manufacturer's warranty, also plays an important role in attracting more customer attention in a market with severe brand competition (Chien, 2005). A good example of an excellent default warranty plan is Hyundai's, which offers a 5 year/60,000 mile bumper-to-bumper and 10 yr/100,000 mile powertrain protection warranty on all of its cars sold in the USA. Consequently, one challenge that a durable product manufacturer must resolve is the determination of the optimal combination of a default base after-sales service plan and the optional after-sales service plan. This is done through an analysis of the trade-off between a profit gain from the optimal plan and a sale gain from the base warranty.

1.1. Supply chain and after-sales service

Offering adequate after-sales service to customers has become a major generator of revenue, profit, and competency in modern industries (Cohen et al., 2006; Cohen and Kunreuther, 2007). At the same time, after-sales services, such as warranties, cost firms a substantial amount of money. Many scholars have explored

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after-sales service from various viewpoints, such as spare parts supply chain, warranty, customer relationship, and so on. However, little research has been published so far on competition between the after-sales service offered by retailers and that offered by manufacturers taking both customer satisfaction and profit maximization into consideration.

In this paper, the objective is to analyze whether the after-sales service that both a retailer and a manufacturer offer to maximize their profit is equivalent to the after-sales service that would most satisfy customers, considering the balance between a default after-sales service and an optional after-sales service for a two-stage supply chain of a manufacturer and a retailer. We note that the equilibrium service level can be understood as that determined from an operations perspective, and the optimal service level in terms of customer satisfaction is that determined from the marketing perspective. In a sense, we analyze the marketing–operations interface in a two-stage supply chain of durable goods and electronics from the view of after-sales service plans. Our model sets up two customer segments: one is the group of customers who use only the base after-sales service offered by a manufacturer (hereafter Segment 0), and the other is the group of customers who apply and pay for an optional after-sales service offered by a retailer in addition to the manufacturer's base after-sales service (hereafter Segment 1). Note that a warranty, as an optional after-sales service with extra payment, is also called a *pro rata warranty* (Matis et al., 2008). That is, the manufacturer's base after-sales service plan influences the purchase of every customer, while the retailer's optional after-sales service plan has an impact on only customers belonging to Segment 1. Fig. 1 is a schematic representation of our modeling framework.

Using the aforementioned supply chain structure, we explore the following business cases:

Case 1. *Nash solution (base model)*: How can both supply chain members, a retailer and a manufacturer, determine the optimal after-sales service level when the service level is determined by the two firms simultaneously, the after-sales services being additive, and there being no interaction between them? We apply a Nash game to tackle this question.

Case 2. *Global optimization*: What will be the optimal after-sales service plan when we consider an integrated model where a manufacturer and a retailer are aggregated and only one decision maker controls the entire system?

Case 3. *Stackelberg solution*: Assuming that manufacturers make an after-sales service decision first, and then retailers make their after-sales service decision next, what will be the optimal after-sales service level for the system? We apply a manufacturer-leader Stackelberg game to solve this question.

Case 4. *Price-sensitive optional service plan*: In the base model, demand for an optional after-sales service plan depends on level of the optional service, not on additional charge for the option. However, we relax this price independency and explore the effect of payment for an optional plan on after-sales service decision.

Case 5. *Interaction between two service plans*: The base model assumes that the manufacturer's warranty and the retailer's optional service plan have no interaction effect. We also investigate the effect of interaction between two after-sales service plans on equilibrium decision.

We also compare the results of Cases 2–5 with that of Case 1 (i.e., the base model). In addition, we propose several managerial insights and implications based on the obtained analytical results.

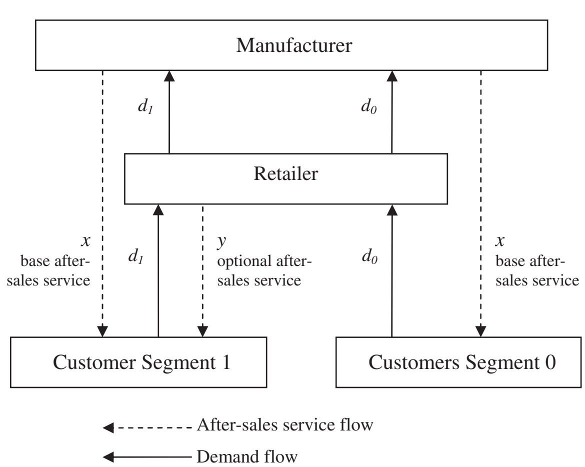

Fig. 1. Supply chain structure.

2. Literature review

In modern business in mainly durable goods, such as computers, automobiles, electric appliances, and construction equipment, after-sales service is regarded as a key revenue generator and a main competitive differentiator. Many scholars have modeled after-sales services. Cohen and Lee (1990) discuss the importance of excellent after-sales service with regard to spare parts, citing two cases: one in the computer industry and the other in the automobile industry. Cohen and Whang (1997) applied a product life-cycle model to study the relationship between product prices and after-sales service levels. In the model of Cohen and Whang (1997), a customer can obtain after-sales service only from either the manufacturer or an independent service shop. However, our model allows a customer to obtain simultaneously two after-sales services, one from the product's manufacturer and the other from the retailer. Interviewing representatives at leading manufacturers, Auromo and Ala-Risku (2005) identify that adequately and simultaneously managing demand for industrial services and supply network structure contributes to integrated and value-added services to customers. Service guarantees can be divided into two categories: economic payout and noneconomic payout. Baker and Collier (2005) define a systematic quantitative model that can determine for business the best portfolio of economic and noneconomic service guarantees. Smith and Eroglu (2009) empirically examine customer's evaluation on various off-site customer service contact methods, such as fax, email, and telephone. They develop a versatile off-site customer service scale that can be applied for various service encounter settings. Kameshwaran et al. (2009) analyze product-service bundling and pricing for a complex durable product that will more likely be maintained than replaced. Their game theoretic model shows how a firm should offer an after-sales program: offering product only, product and service independently, or product and service bundled. Analyzing the panel data over various product categories, Chen et al. (2010) explore the impact of extended service contracts on retail business of durable products. Their results show how an extended service purchase is influenced by hedonic/utilitarian value of products, marketing action, and consumer's characteristics.

Supersaturation of marketing efforts on customer response is a commonly experienced occurrence. For example, the majority of Internet users consider spam – unsolicited electronic junk mail from business that promote or advertise goods and services – so annoying and bothersome that the US government passed an anti-spam law in 2003 (Haag et al., 2007). On the other hand, marketing efforts on the part of Southwest Airlines and JetBlue Airways are frequently cited as successful examples of airlines applying a no-frills service policy. However, very little academic research has been conducted on supersaturation of marketing efforts, especially on after-sales service. A paper by Waid et al. (1956) that studied how the number of sales calls would influence business returns could be among the oldest academic articles that discuss marketing effort supersaturation. Empirical research by Mukherjee and Hoyer (2001) showed that addition of novel attributes to high-complexity products, such as computers, can decrease a customer's evaluation of the product, even though it was believed that adding novel attributes to a product would improve product evaluation.

Several papers explore the relationship between a product's quality and warranty. Balachander (2001) explained that the correlation between quality and warranty differs for an established incumbent product and a new-entrant product. Using a game theoretic model, Balachandran and Radhakrishnan (2005) studied warranty contracts between a parts supplier and a manufacturer with respect to quality information from an incoming inspection and external failures. Including the manufacturer's cost for warranties and the customer's cost due to product failure, Chien (2005) developed an analytical model to consider the optimal warranty period and the optimal replacement age after warranty expiration. Matis et al. (2008) numerically analyzed average profit and non-renewing base and *pro rata warranties* and determined the optimal pricing, warranty length, and repair option selection. Jack and Murthy (2006) determined the optimal warranty policy when a customer has an option to decide the timing to take out an extended warranty and the length it will last. Assuming a case where a manufacturer sells directly to customers, a game theoretic model proposed the optimal warranty strategies. Empirical research examined the existing literature on warranty theory (Oumlil, 2008). Oumlil presented how interaction between producers and buyers determines the optimal price/quality/warranty combination for the product, and suggested several managerial implications, for example standardizing the warranty becomes more difficult as the product line becomes more diverse. Huang and Yen (2009) developed a warranty model that includes effects of both age and usage on deteriorating products and recovery of the product condition due to preventive maintenance. Their analytical results offer a managerial direction for a manufacturer on building a proper warranty program. Zhou et al. (2009) developed mathematical models to study a joint dynamic pricing and warranty policy for a product with a fixed product life for a manufacturer and to demonstrate the benefits of dynamic pricing and warranty. Hartman and Laksana (2009) investigate, using optimization models, the consumer's selection of extended warranty, the warranty provider's optimal pricing decision, and the design of a menu of warranty contracts for heterogeneous consumers. Using a dynamic profit-maximization model, Lin et al. (2009) numerically present optimal control paths for price, production rate, and warranty length and then give an insight on the complex interdependencies of firms' business functions. Chu and Chintagunta (2009) consider the role of a base warranty program in server markets. Their empirical analysis presented that warranty gives benefits to manufacturers, intermediate firms in a supply chain, and customers. Also, they showed the value of offering a menu of warranty policies to customers and price discrimination by offering non-uniform warranties. Although the authors refer to both base warranty and extended warranty (equivalent to the optional warranty in our paper), their analysis focused only on the base warranty.

3. Models

Two segments exist in our model. Segment 0 contains customers who use only the base after-sales warranty offered by a manufacturer. Usually, such a base warranty is by default and free or, more precisely, the cost of the default after-sales service is included in the retail price such that customers do not pay any extra charge for service when they buy the product. Segment 1 contains customers who purchase an optional after-sales service plan by paying an extra charge. We believe that establishing the two segments as such is realistic. This is because our observations of shopping behavior at computer and electronics shops show that some customers (e.g., those who are very computer-savvy or have a limited budget) will decline the shop employee's offer to apply for an additional after-sales service plan, such as a 3-year retailer's warranty. Instead, such customers use only the basic after-sales service offered by the manufacturer, such as a 1-year manufacturer's warranty. We assume that payment for an optional plan is $r>0$. We assume that a retail price p, wholesale price w, and a unit production cost c are exogenous. Note that an exogenous retail price is not uncommon for durable products such as home electronics and PCs, since most retail chains adopt a *price-matching strategy* where one store will reduce its retail price as low as a competitor's if customers claim that a rival shop is selling the same product at a cheaper price. Table 1 lists all the notations and symbols in our models.

3.1. Demand functions

We assume that an after-sales service level determines the demand for the product. Our key assumption is the existence of after-sales service supersaturation; that is, there exists an optimal service level that will maximally satisfy customers. In other words, the offer of excessive service will discourage customers from buying the products. Effects of such supersaturation are commonly observed in the service and retail industry. For example, there is such an abundance of fast food restaurants and budget hotel chains because many people just want an average level of service quality. Following the explanation on supersaturation by Hanssens et al. (2001) and considering

Table 1
Symbols and superscripts.

Symbols	
x	Basic after-sales service level determined by a manufacturer
y	Optional after-sales service level determined by a retailer
$\bar{x}$	Optimal after-sales service level for a manufacturer ($\bar{x}>1$)
$\bar{y}$	Optimal after-sales service level for a retailer ($\bar{y}>1$)
k_M	Coefficient for manufacturer's cost of offering after-sales service
k_R	Coefficient for retailer's cost of offering after-sales service
a_0	Base market size for Segment 0
a_1	Base market size for Segment 1
b_0	Sensitivity for after-sales service for Segment 0
b_1	Sensitivity for after-sales service for Segment 1
p	Retail price ($p>w$)
w	Wholesale price ($w>c$)
c	Unit production cost ($c>0$)
r	Unit profit from an optional after-sales service ($r>0$)
d_0	Demand of Segment 0
d_1	Demand of Segment 1
$\pi_M(x\|y)$	Manufacturer's profit
$\pi_R(y\|x)$	Retailer's profit
β	Coefficient of positive interaction of after-sales services between a manufacture and a retailer ($0\le\beta\le1$)
γ	Effect of extra charge on demand for an optional after-sales service ($0\le\gamma$)
$A(x)$	$[k_M-2b_0(w-c)(\bar{x}-x)][(p+r-w/k_M)]$
Superscripts	
*	Nash solutions for the base case (Case 1)
0	Global solution for the centralized case (Case 2)
**	Stackelberg solutions (Case 3)
+	Nash solutions for the price sensitive case (Case 4)
I	Nash solution for the interaction case (Case 5)

mathematical tractability, we model this service saturation using a quadratic demand function with respect to the level of after-sales service, where the optimal service levels are represented by $\bar{x}$ and $\bar{y}$. The values of $\bar{x}$ and $\bar{y}$can be estimated by firms when conducting consumer behavior surveys or interviews. Without loss of generality, we assume $\bar{x}>1$ and $\bar{y}>1$.

As mentioned above, another key assumption is the existence of two customer segments. As a result, we have two demand functions. Demand for Segment 0 is defined as

$$d_0=d_0(x)\equiv a_0+b_0x(2\bar{x}-x).$$

In Segment 1, customers pay a charge ($r>0$) for the retailer's optional after-sales service plan. We assume that customers recognize the sum of the manufacturer's basic after-sales service (x) and the retailer's optional after-sales service (y) as the total perceived service level, which determines demand. Thus, demand for Segment 1 is defined as

$$d_1=d_1(x,y)=a_1+b_1(x+y)(2\bar{x}+2\bar{y}-x-y).$$

The establishment of two distinct customer segments is common practice in analytical research in management science. For example, revenue management studies often assume two segments of customers: business passengers and leisure passengers. For our study, we first analyze the simple case in which the price of the optional after-sales service does not influence demand and the two after-sales service levels, x and y, are additive. However, we later relax several assumptions of the base model. Sections 4.2–4.5 address the equilibrium service plans in the more generalized situation.

3.2. Profit functions

In our model framework, retail price, wholesale price, and unit production cost are assumed exogenous for analytical tractability and to focus on the effect of after-sales service. Thus, a manufacturer's unit profit is determined as $w-c$. In addition, cost to the manufacturer for after-sales service is assumed to be proportional to level of after-sales service for tractability. As a result, profit function for the manufacturer will be composed of revenue from total sales from the two segments (i.e., $(w-c)(d_0(x)+d_1(x,y))$) and after-sales service cost (i.e., k_Mx). That is, profit function for the manufacturer will be

$$\pi_M(x|y)=(w-c)\{a_0+b_0x(2\bar{x}-x)+a_1+b_1(x+y)(2\bar{x}+2\bar{y}-x-y)\}-k_Mx.$$

For the retailer, we assume that the retailer gains an additional profit r if a customer adopts an optional service plan. Hence, profit function for the retailer contains three terms. The first term, $(p-w)d_0(x)$, represents profit from the customers who do not apply for the retailer's optional after-sales plan. The second term, $(p+r-w)d_1(x,y)$, represents profit from customers who do apply for the retailer's optional after-sales plan, and the last term, k_Ry, represents cost generated by offering the after-sales service plan of y. Consequently, profit function for the retailer is defined as

$$\pi_R(y|x)=(p-w)\{a_0+b_0x(2\bar{x}-x)\}$$
$$+(p+r-w)\{a_1+b_1(x+y)(2\bar{x}+2\bar{y}-x-y)\}-k_Ry.$$

3.3. Performance measure

Our analysis has two *best* after-sales service plans. One is an *equilibrium after-sales service plan* (or an *equilibrium solution*) that can be determined by solving a game theoretic model between a manufacturer and a retailer. The other is an *optimal after-sales service plan* (or an *optimal solution*) that can maximally satisfy customers and is assumed to be exogenously given in our analysis. Note that Case 2 is not a game theoretic model, but we still call the global optimal solution the equilibrium after-sales service level. In a sense, the equilibrium solution is the best after-sales service decision from an operations perspective, while the optimal solution is the best after-sales service decision from a marketing perspective. Hence, in our model, as research on the marketing–operations interface, a situation where the equilibrium solution is equivalent to the optimal solution is considered an *ideal* situation for the after-sales service decision.

Note that we set up two ideals: *Ideal Type* 1 is that the equilibrium solution is the same as the optimal solution for each entity, while *Ideal Type* 2 is defined as the sum of two equilibrium service levels being the same as the sum of two optimal service levels. Obviously, Ideal Type 2 is weaker than Ideal Type 1. For Ideal Type 2, we measure the difference between the ideal solution and the optimal solution using a Manhattan distance, and this distance is used to measure quality of the total after-sales service offered by both a manufacturer and a retailer. In Fig. 2, the value of $\bar{x}+\bar{y}-x-y$, represented by the inverse-L-shaped bold line, is the measure of service quality that is used to evaluate Ideal Type 2. Clearly, a shorter line indicates better quality. Also, in Fig. 2, a line A–A' represents all the points where the sum of x and y equals $\bar{x}+\bar{y}$. In our framework, any point on the A–A' line can achieve Ideal Type 2, which guarantees the same utility for customers as the optimal point $(\bar{x},\bar{y})$. On the other hand, Ideal Type 1 can be attained only if the equilibrium after-service levels are exactly same as the optimal point $(\bar{x},\bar{y})$.

A final note on after-sales service in our research is that, generally, our "after-sales service" includes any type of service available after the purchase of a product, such as a free warranty, a telephone or email helpdesk, training sessions, tutorial materials, and so forth. Yet, technically, a practical way to understand quality of "after-sales service" is to consider length of warranty support.

4. Model analysis

4.1. Nash solution for the non-interaction case

Under the aforementioned assumptions, we formulate profit functions as follows.

For a retailer

$$\pi_R(y|x)=(p-w)\{a_0+b_0x(2\bar{x}-x)\}$$
$$+(p+r-w)\{a_1+b_1(x+y)(2\bar{x}+2\bar{y}-x-y)\}-k_Ry \quad (1)$$

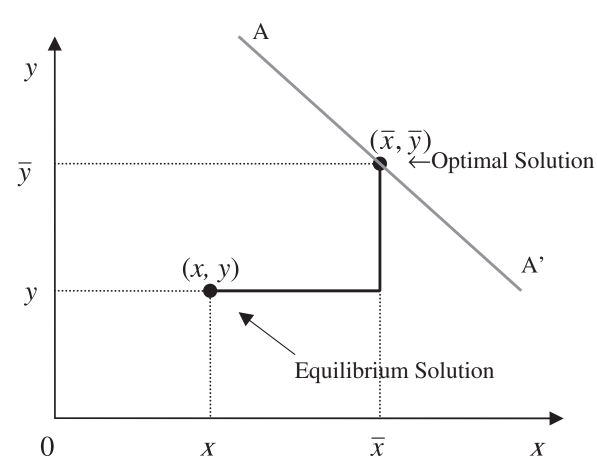

Fig. 2. Measure of service quality.

and for a manufacturer

$$\pi_M(x|y) = (w-c)\{a_0 + b_0x(2\bar{x}-x) + a_1 + b_1(x+y)(2\bar{x}+2\bar{y}-x-y)\} - k_M x. \quad (2)$$

Proposition 1 shows the Nash solution.

Proposition 1. *Nash solution for equilibrium after-sales service levels*:

$$x^* = \bar{x} + \frac{k_R}{2b_0(p+r-w)} - \frac{k_M}{2b_0(w-c)},$$

$$y^* = \bar{y} - \frac{k_R(b_0+b_1)}{2b_0b_1(p+r-w)} + \frac{k_M}{2b_0(w-c)}.$$

Proof: All proofs are presented as an Appendix.

For Proposition 1, it is easily shown that when a decision ignores variable cost of offering after-sales services (i.e., $k_R = k_M = 0$), we can achieve $x^* = \bar{x}$ and $y^* = \bar{y}$. In a sense, if a sales department considers only customer satisfaction when creating an optimal after-sales service plan, then the Nash solution for the after-sales service plan will be equivalent to the optimal service level for both manufacturer and retailer. However, inclusion of the cost factor makes the analysis more realistic. Next, Proposition 2 analyses the conditions under which the Nash solution including a cost factor will be identical to the optimal service level.

Proposition 2. *Difference between Nash solution and optimal after-sales service levels*:

(a) *if and only if* $b_0 = 0$, *then* $x^* = \bar{x}$ *and* $y^* = \bar{y}$,
(b) *if* $b_0 > 0$, *then* $x^* + y^* = \bar{x} + \bar{y} - \frac{k_R}{2b_1(p+r-w)} < \bar{x} + \bar{y}$.

What we know from Proposition 2 is three-fold. Firstly, the Nash solution of the after-sales service competition results in the optimal service level only in the case in which the base after-sales service plan does not affect demand for Segment 0, i.e., the customers who do not pay for an optional plan. A free manufacturer's warranty was originally intended to be used as a product's selling point. However, once all brands in a category offer a similar base warranty plan, the promotional effect of such a warranty is lost. In fact, almost all new PCs are sold with a 1-year manufacturer's warranty. Yet, from our observations, PC buyers regard such a base warranty as a natural right and, as a result, availability of a free 1-year warranty hardly influences purchase decisions.

Secondly, once the base service plan influences demand of Segment 0 (i.e., $b_0 > 0$), the after-sales service level obtained as a Nash solution will differ from the optimal level. Proposition 2(b) shows that the sum of equilibrium after-sales service levels is lower than that of the actual optimal levels of after-sales service. In other words, if both a manufacturer and a retailer decide after-sales service levels independently, then levels of after-sales service actually offered by the firms are underestimated. The Nash solution can be interpreted as the most profitable after-sales service plan under competition. However, discrepancy between the Nash solution and the optimal service level might generate the risk in the long run that customer satisfaction and, furthermore, brand equity of the product will be undermined. Note that developing a good life-long relationship with customers is a business issue that has recently been discussed.

Thirdly, the difference between equilibrium after-sales service level and actual optimal after-sales service levels is determined by the parameters, k_R, b_1, p, r, and w, which are retailer's factors, roughly speaking. Hence, it may be safe to say that this difference is untouched by the manufacturer's business environment. However, the retailer's business policy might influence the discrepancy.

4.2. Centralized model

As the counterpart to the decentralized supply chain discussed in Section 4.1, here we explore the centralized supply chain in which one decision maker controls both manufacturer and retailer. The profit function of the centralized model is defined as

$$\Pi(x,y) = (p-c)\{a_0 + b_0x(2\bar{x}-x)\} - k_M x + (p+r-c)\{a_1 + b_1(x+y)(2\bar{x}+2\bar{y}-x-y)\} - k_R y. \quad (3)$$

Therefore, the optimal after-sales service plan can be determined by solving (3) with respect to x and y simultaneously.

Proposition 3. *Optimal after-sales service level in centralized model*:

$$x^0 = \bar{x} + \frac{k_R - k_M}{2b_0(p-c)},$$

$$y^0 = \bar{y} - \frac{k_R - k_M}{2b_0(p-c)} - \frac{k_R}{2b_1(p+r-c)}.$$

Note that a superscript "0" represents the solution for the centralized model.

Proposition 4. *Behavior of the solution for centralized model*:

(a) *Suppose* $b_0 = 0$. *Then* $x^0 \neq \bar{x}$ *and* $y^0 \neq \bar{y}$ *unless* $k_M = k_R$.
(b) $x^* + y^* < x^0 + y^0 = \bar{x} + \bar{y} - \frac{k_R}{2b_1(p+r-c)} < \bar{x} + \bar{y}$.

We assume that a shorter Manhattan distance between optimal service level (i.e., $(\bar{x},\bar{y})$) and equilibrium service level (i.e., (x^*,y^*)) on the x–y plane indicates a better equilibrium service level. From this perspective, Proposition 4(b) proves that the best solution in the centralized system outperforms the Nash solution in the decentralized system. However, in the decentralized system, there is a chance that the Nash solution will be the optimal service level (achieved when $b_0 = 0$ from Proposition 2). However, interestingly the condition $b_0 = 0$ does work well in the centralized system only in the case of $k_M = k_R$, i.e., cost performance for after-sales service is equivalent for a manufacturer and a retailer. It is natural that the promotion cost situation differs between the two firms. That is, there is a good chance that the condition $b_0 = 0$ usually does not work in the centralized system. The finding in Proposition 4 can be interpreted as indicating that the choice between the centralized and the decentralized systems depends on customer behavior. If customers are insensitive to the base after-sales service offered by a manufacturer, maintaining a decentralized system will be more reasonable (as long as business goals focus on managing the after-sales service level offered as close to optimal as possible).

4.3. Case with a manufacturer-leader Stackelberg model

It is not uncommon that a retailer's decision about after-sales service depends on that offered by a manufacturer. In fact, a computer chain usually offers customers an optional, say, 3-year warranty in addition to the manufacturer's basic 1-year warranty. Therefore, this section considers the optimal after-sales service level by assuming that a manufacturer is a decision leader who first decides their level of after-sales service and a retailer is a decision follower, who determines an after-sales service level depending on the service level offered by the manufacturer. We formulate a Stackelberg game to represent this sequential decision. Proposition 5 shows the solution.

Proposition 5. *Stackelberg solution*:

$$x^{**}=\bar{x}-\frac{k_M}{2b_0(w-c)},$$

$$y^{**}=\bar{y}+\frac{k_M}{2b_0(w-c)}-\frac{k_R}{2b_1(p+r-w)}.$$

*Note that superscript "**" represents the Stackelberg solution.*

Next, we explore the relationship between the Stackelberg solution and the optimal after-sales service level.

Proposition 6. *Difference between Stackelberg solution and optimal after-sales service level*:

(a) *under our assumptions,* $y^{**}=\bar{y}$ *might be achieved when* $(k_M/2b_0(w-c))=(k_R/2b_1(p+r-w))$, *while* $x^{**}=\bar{x}$ *is impossible*;
(b) *the condition* $b_0=0$ *does not achieve* $x^{**}=\bar{x}$ *and* $y^{**}=\bar{y}$;
(c) $x^{**}+y^{**}=\bar{x}+\bar{y}-\frac{k_R}{2b_1(p+r-w)}=x^*+y^*<\bar{x}+\bar{y}$.

Implications from Proposition 6 are four-fold. Firstly, the $b_0=0$ resolution for the base Nash case does not work well in the Stackelberg case. Secondly, while there is a chance that at least the base service level from the Stackelberg model will be equivalent to the optimal base service level, the condition for achieving this equivalency is not always guaranteed. Thirdly, and most importantly, even when a retailer creates an after-sales service policy after the manufacturer's base service policy has been decided, the total level of after-sales service becomes lower than the optimum level that would maximally satisfy customers. In other words, we have shown that an after-sales service level decision decided rationally based on profit maximization differs from the optimal after-sales service level that can maximize customer satisfaction. Finally, the distance between the optimal service plan (i.e., $(\bar{x},\bar{y})$) and the obtained service plan from the Stackelberg equilibrium (i.e., (x^{**},y^{**})) is identical to the distance in the Nash case. In other words, shifting the Nash case to the Stackelberg case (and vice versa) merely reallocates the effort of after-sales service between a manufacturer and a retailer rather than improving system-wide customer satisfaction due to the offer of after-sales service plans. Here Fig. 3 graphically summarizes Proposition 6.

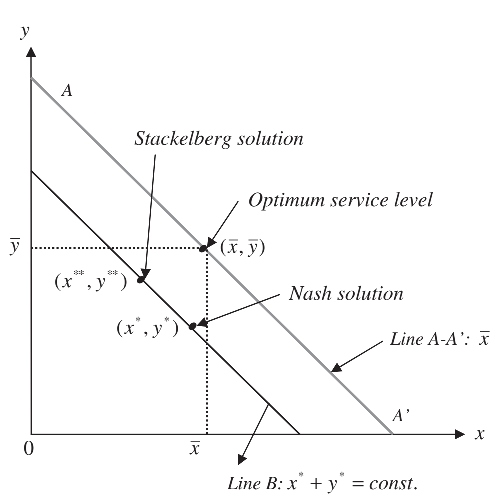

Fig. 3. Relationship among the solutions.

4.4. Case in which segment 1 is influenced by the extra charge for an optional service

The implicit assumption so far is that Segment 1 is influenced only by level of an optional after-sales service and not by extra charge for additional service. The current section relaxes this assumption. That is, a higher price for the optional after-sales service will discourage the demand for the optional service. In this section, the demand for Segment 1 that is negatively influenced by the extra charge is determined by

$$d_{1+}=d_1(x,y)=a_1+b_1(x+y-\gamma r)(2\bar{x}+2\bar{y}-x-y+\gamma r), \quad (4)$$

where γ represents the negative impact of extra charge to customers on the demand for the optional after-sales service ($0\le\gamma$). It is easy to show that the value of y that satisfies the first-order condition in (4) is greater than that in (1), which means that, as customers belonging to Segment 1, they want a higher level of optional service if they have to pay a larger extra charge. In other words, if the level of optional service remains the same, an extra charge discourages customers from choosing the optional service. As a result, (4) includes the negative effect of paying an extra charge on the demand for the optional service. Note that we assume that the amount of extra charge for an additional service, r, is a parameter and not a function of level of the optional after-sales service level, y. This is because our observations indicate that level of after-sales service is independent of both price of the products and amount of extra charge for the optional service. The superscript "+" indicates the case with the negative effect of extra charge. In this setting, profit functions for the manufacturer and the retailer are defined as (5) and (6) for a manufacturer and a retailer, respectively:

$$\pi_M^+(x|y)=(w-c)\{a_0+b_0x(2\bar{x}-x)+a_1+b_1(x+y-\gamma r)(2\bar{x}+2\bar{y}-x-y+\gamma r)\}-k_Mx, \quad (5)$$

$$\pi_R^+(y|x)=(p-w)\{a_0+b_0x(2\bar{x}-x)\}+(p+r-w)\{a_1+b_1(x+y-\gamma r)(2\bar{x}+2\bar{y}-x-y+\gamma r)\}-k_Ry. \quad (6)$$

When taking into account the negative effect of extra charge, the Nash equilibrium of after-sales service plans is modified as follows.

Proposition 7. *Nash solution when an extra charge negatively influences demand*:

(a) $$\begin{bmatrix}x_+\\y_+\end{bmatrix}=\begin{bmatrix}\bar{x}\\\bar{y}\end{bmatrix}+\begin{bmatrix}k_R/2b_0(p+r-w)-k_M/2b_0(w-c)\\-k_R(b_0+b_1)/2b_0b_1(p+r-w)+k_M/2b_0(w-c)\end{bmatrix}+\begin{bmatrix}0\\\gamma r\end{bmatrix},$$
(b) $x^+=x^*$,
(c) $y^+>y^*$ *if* $\gamma>0$ *and* $y^+=y^*$ *if* $\gamma=0$,
(d) *The condition* $b_0=0$ *achieves* $x^+=\bar{x}$ *and* $y^+=\bar{y}$ *only when* $(k_M/(w-c))=(k_R/(p+r-w))$.

Propositions 7(b) and (c) show that the equilibrium service level for the retailer becomes smaller only when paying an extra charge negatively affects customer demand for an additional service, while equilibrium service level for the manufacturer is not influenced. This finding is preferable for the manufacturer, as the manufacturer does not have to revise its after-sales service policy even though additional charge from the retailer influences the demand for the optional service and the retailer revises its after-sales service plan to include a charge. In addition, we know from Proposition 7(d) that the strategy setting $b_0=0$, which can make the equilibrium service plan equivalent to the optimal service plan, does not work unless $(k_M/w-c)=(k_R/p+r-w)$, that is, the ratio of cost to revenue situation is the same between the two firms. In other words, if the cost–revenue relationship with respect to after-sales service differs between a retailer and a manufacturer (e.g., one firm has a much higher margin than the other), the strategy setting $b_0=0$ will not work anymore; the equilibrium service level will differ from the optimum level.

4.5. *Case in which retailer's optional after-sales service plan gains additional profit*

Thus far, we have assumed that the manufacturer's base after-sales service plan and the retailer's optional after-sales service plan are additive and have no interaction effects. This section relaxes this assumption. That is, here we assume that demand is influenced, either negatively or positively, by the interaction term of the two after-sales plans, termed hereafter the *interaction case*. A parameter β represents such an interaction between two after-sales services. As a result, after adding the interaction term, profit functions (1) and (2) can be modified as follows.

For a manufacturer, (1) becomes

$$\pi_M^I(x|y)=(w-c)\{a_0+b_0x(2\bar{x}-x)+a_1 + b_1(x+y+\beta xy)(2\bar{x}+2\bar{y}+2\beta\bar{x}\bar{y}-x-y-\beta xy)\}-k_Mx. \quad (7)$$

For a retailer, (2) becomes

$$\pi_R^I(y|x)=(p-w)\{a_0+b_0x(2\bar{x}-x)\}+(p+r-w) \times\{a_1+b_1(x+y+\beta xy)(2\bar{x}+2\bar{y}+2\beta\bar{x}\bar{y}-x-y-\beta xy)\}-k_Ry. \quad (8)$$

Note that the superscript "I" represents the interaction case. Proposition 8 below presents the Nash solution for the interaction case.

Proposition 8. *Nash solution for interaction case*:

(a) *as* β *increases, both* x^I *and* y^I *increase*,
(b) $x^I=\bar{x}$ *and* $y^I=\bar{y}$ *cannot be attained, not even when* $b_0=0$.

Note that x^I *and* y^I *represent the Nash solutions for the interaction case*.

Proposition 8(b) shows that the strategy setting $b_0=0$ that works in Case 1 would be useless to achieve the ideal outcome once an interaction term is included in the model.

5. Numerical examples and business implications

Thus far, we have analyzed the basic model in Section 4.1 and several extended models: a centralized model (Section 4.2), a manufacturer-leader Stackelberg model (Section 4.3), a case of price-sensitive Segment 1 (Section 4.4), and a case with interaction (Section 4.5). In our research, the business has two objectives: (a) to obtain the equilibrium after-sales service plan that maximizes profit (and is stable) and (b) to offer after-sales service plans as close to the customers' expectations as possible. In our analysis, the answer for the first objective can be obtained as the solution to the first-order conditions of profit functions, and the second objective is measured as the distance between obtained solution and optimal after-sales service level (i.e., $(\bar{x},\bar{y})$).

The first half of Section 5 uses several numerical examples to examine our analytical discussion. For the numerical examples in this section, the parameters are set as $a_0=100$, $a_1=100$, $b_0=3$, $b_1=3$, $p=50$, $w=20$, $c=10$, $r=5$, $k_M=50$, $k_R=40$, $\gamma=0.1$, $\beta=0.04$, $\bar{x}=10$, and$\bar{y}=20$. Firstly, Table 2 and Fig. 4 present equilibrium after-sales service levels among the five cases, and optimal after-sales service levels. Table 2 shows that the sum of equilibrium levels of the manufacturer and the retailer are lower than the optimal level. Fig. 4 illustrates that all equilibrium service levels have been distributed around the optimal service levels ($\bar{x}=10$ and $\bar{y}=30$).

In Fig. 4, the gray line connecting points A and A' indicates the combination of x and y, the sum of which is equivalent to 30 ($=\bar{x}+\bar{y}$). If the equilibrium is located below (above) the A–A' line, one can conclude that total after-sales service is less than (more than) the optimal level. In the numerical examples, the equilibrium after-sales service levels in Cases 1–3 become a less-than optimal after-sales level of $\bar{x}+\bar{y}=30$, while equilibrium service levels in Cases 4 and 5 are above the $\bar{x}+\bar{y}=30$ line. This observation shows that once an additional charge on an optional after-sales service negatively influences customers' purchase decisions or once after-sales service plans offered by a manufacturer and a retailer interact positively, there is a chance that the sum of the service level will exceed the sum of optimal service levels (i.e., $\bar{x}+\bar{y}$). In other words, if a business goal is to offer an amount of after-sales services that is equivalent to the optimal level from a customer satisfaction perspective, there is a way to

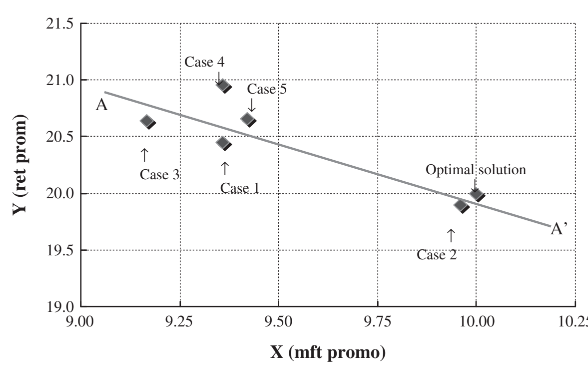

Fig. 4. Comparison of equilibrium promotion levels in the five cases.

Table 2
Comparison of equilibrium promotion levels in the five cases.

Case		Equilibrium x (manufacturer's promotion level)	Equilibrium y (retailer's promotion level)	Sum of equilibrium x and y
Case 1	*Nash case*	9.36	20.45	29.81
Case 2	*Centralized case*	9.96	19.89	29.85
Case 3	*Stackelberg case*	9.17	20.64	29.81
Case 4	*Charge effect case*	9.36	20.95	30.31
Case 5	*Interaction case*	9.42	20.66	30.08
Optimal promotion level		10	20	30

achieve it. Of course, we shall notice that satisfying customers by offering the most adequate after-sales service plan does not guarantee maximized profit for the supply chain or each member belonging to the chain. Which among the five cases in our analysis is the best depends on the situation.

Secondly, Table 3 demonstrates the sensitivity of the value β in the Nash solution. We know that as the value of β increases (that is, as two after-sales more positively influence the customer's quality perception), equilibrium after-sales service levels increase for both the retailer and the manufacturer.

Thirdly, Fig. 5 visualizes the effect of a change in the retailer's promotion cost parameters k_R on the Nash solution when k_M is fixed as 50. In this numerical example, k_R is assumed to be an equivalent value and changed from 0 to 90. Note that Fig. 5 shows that the equilibrium solutions when $k_R=40$ will be equivalent to the Nash equilibrium service levels in Case 1 in Table 3 (i.e., $x^*=9.36$ and $y^*=20.45$), and as cost increases, x^*decreases while y^* increases.

Table 3
Effect of β on equilibrium service levels of Case 1.

β	x^*	y^*	β	x^*	y^*
0.000	9.357	20.452			
0.004	9.365	20.484	0.056	9.436	20.701
0.008	9.373	20.512	0.060	9.439	20.710
0.012	9.380	20.537	0.064	9.443	20.718
0.016	9.387	20.560	0.068	9.446	20.725
0.020	9.393	20.580	0.072	9.449	20.732
0.024	9.399	20.599	0.076	9.452	20.739
0.028	9.404	20.616	0.080	9.455	20.745
0.032	9.410	20.632	0.084	9.458	20.751
0.036	9.415	20.646	0.088	9.460	20.756
0.040	**9.419**	**20.659**	0.092	9.463	20.761
0.044	9.424	20.671	0.096	9.465	20.766
0.048	9.428	20.682	0.100	9.468	20.770
0.052	9.432	20.692	0.104	9.470	20.774

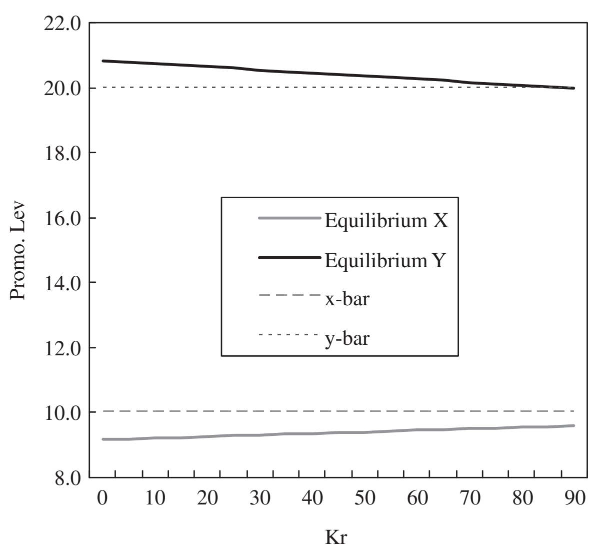

Fig. 5. Effect of promotion cost k_R on the equilibrium of Case 1 when $k_M=50$.

In the second half of Section 5, we have proposed several business implications. Below, summarizing our findings in Section 4, Table 4 lists our answers about whether or not the equilibrium solution (i.e., the best from an operations perspective) is equivalent to the optimal service levels (i.e., the best from a marketing perspective).

We set up two ideal situations: Ideal Type 1 in which the equilibrium solution (i.e., the best after-sales service from an operations perspective) is the same as the optimal solution (i.e., the best after-sales service from a marketing perspective) for either a retailer or a manufacturer, and Ideal Type 2, in which the sum of the two equilibrium solutions is equivalent to the sum of the two optimal solutions. In Table 4, Columns 2 and 3 show the answers for Ideal Types 1 and 2, respectively.

Case 1 (i.e., base model) can achieve Ideal Type 1 only when $b_0=0$. However, Ideal Type 1 is not possible in general for the remaining Cases 2–5 (see Column 2a). Column 2b summarizes whether a strategy with $b_0=0$ that is good for Case 1 will also work appropriately for other cases. It is interesting to know that $b_0=0$ does not work adequately in Cases 2 and 5, and some additional condition on costs is required for Cases 3 and 4 to attain Ideal Type 1 setting $b_0=0$. Note that $b_0=0$ means that a basic after-sales service does not influence customers' purchase decision. It is safe to say that principally it is very difficult or impossible to match equilibrium after-sales service levels with the optimal level that customers want the most. In other words, business can rarely or almost never enjoy perfect marketing–operations coordination with respect to after-sales service. Ideal Type 1 can be realized only when a basic after-sales service does not affect customers and service decisions are independently and

Table 4
Summary of the findings.

Column 1	Column 2		Column 3
	Ideal Type 1 is achieved? (each equilibrium solution = each optimal solution)		***Ideal Type 2 is achieved?*** (sum of equilibrium solutions = sum of optimal solutions)
Case 1 (Nash solution base model)	Yes if and only if $b_0=0$		No
	Column 2a ***In general, ideal Type 1 is achieved?***	Column 2b ***Ideal Type 1 is achieved if $b_0=0$***	
Case 2 (integrated model)	No	No	No
Case 3 (manufacturer-leader Stackelberg game)	No	Yes, only when $k_M=k_R$	No
Case 4 (effect of additional charge on demand)	No	Yes, only when $(k_M/w-c)=(k_M/p+r-w)$	Possible
Case 5 (interaction between the two after-sales services)	No	No	Possible

simultaneously decided by a retailer and a manufacturer. However, if the model setting changes, Ideal Type 1 will not be guaranteed anymore. Another interesting finding is that even a centralized supply chain cannot offer an optimal after-sales service plan that can maximally satisfy customers.

Column 3 summarizes the conditions for Ideal Type 2. Only Cases 4 and 5 have a chance to realize Ideal Type 2, while Cases 1–3, which are simpler models, have no opportunity to enjoy Ideal Type 2. It is interesting that Ideal Type 2 can be achievable, although there exists a negative effect of an additional charge for optional after-sales service (see Case 4). Also, comparing Column 2 with Column 3, we know that whether or not Ideal Type 1 is possible does not always match with whether or not Ideal Type 2 is possible. This implies that business must choose an adequate supply chain situation according to what kind of business goal they aim at with respect to on after-sales service. In general, our research shows the complexity and unexpected behavior of the best after-sales service decisions for a two-stage supply chain where a manufacturer offers free basic after-sales service and a retailer offers optional after-sales service with an additional charge. Our analytical discussion and comparison gives new insight on marketing–operations interface issues in supply chain management.

6. Concluding remarks

We have analyzed the interaction between basic after-sales service offered by a manufacturer and optional after-sales service offered by a retailer for durable consumer goods when two customer segments obtain different after-sales service plans: one segment receives only basic after-sales service from the manufacturer and the other purchases an additional service plan offered by the retailer. Our main goal is to explore whether the equilibrium after-sales level that a manufacturer and a retailer decide upon is equivalent to the optimal after-sales service in terms of customer satisfaction. A key feature of our model is the existence of supersaturation with respect to the level of after-sales service. That is, there exists an optimal after-sales level for each segment, and an excessive service offer will degrade the customer's purchase. We first formulated a Nash game model in which each firm determines after-sales service level in order to maximize its profit, assuming that two after-sales service plans have an additive effect on customers' purchases. We then extend the analysis, formulating a centralized model, a manufacturer-leader Stackelberg model, with the model including a negative effect of an additional charge on sales, and a model including a correlation effect, and compared the five different results.

Our analysis highlights the discrepancy between the equilibrium after-sales service level decided by the retailer and the manufacturer and the optimal after-sales service level in terms of customer preference. We analyzed all five cases and found that the equilibrium after-service levels do not guarantee the optimal ones that can perfectly satisfy customers. Equilibrium service levels determined by maximizing the firm's profit (i.e., the best after-sales service from an operations perspective) are generally lower than the optimal levels that satisfy customers the most (i.e., the best after-sales service from a marketing perspective). Interestingly, even a centralized model cannot guarantee such consistency between the two types of service levels under our model framework. Moreover, we recognized that several specific conditions must be set up, for example $b_0=0$ for the Nash model, in order to achieve consistency between the equilibrium service level and the optimal service level for customers. As research on marketing–operations interface, our analytical findings imply that a serious problem might be hidden in the distribution system regarding durable goods and home electronics in which after-sales service plans, such as warranties, play an important role in capturing customer attention and sales.

Further research on the theme of this paper could follow two directions. One would be a technical extension: Our model can be revised to more general assumptions, for example relaxing the quadratic demand function to a general convex demand function, changing the linear after-sales cost function to a nonlinear one, or altering the Euclidian norm used to measure service levels to other standard norms. Another research direction is to expand our research to business situations that could be common in a durable goods industry. For example, the product life of so-called hi-tech products (e.g., mobile computers, cellular phones) is very short so that a customer tends to replace an existing item to a new model instead of claiming maintenance and warranty support. Hence, the optimal warranty and after-sales service competition under quick obsolescence is worth discussing. Also, it is true that some electronic products (e.g., printers and copiers) are not as durable as other consumer electric appliance (e.g., TV sets, refrigerators, and air conditioners). In fact, maintenance is a main revenue generator in the printer and copier business. Another interesting research agenda might explore how a manufacturer or a retailer should manage after-sales, maintenance, and warranty policies over product categories of differing reliability level.

Appendix A

A.1. Proof of Proposition 1

FOC for (1):

$$\frac{\partial \pi_R(y|x)}{\partial y} = (p+r-w)2b_1(\bar{x}+\bar{y}-x-y)-k_R \underset{set}{=} 0.$$

Then

$$y^* = \bar{x}+\bar{y}-x-\frac{k_R}{2b_1(p+r-w)}. \tag{9}$$

FOC for (2):

$$\frac{\partial \pi_M(x|y)}{\partial x} = (w-c)\{2b_0(\bar{x}-x)+2b_1(\bar{x}+\bar{y}-x-y)\}-k_M \underset{set}{=} 0.$$

Then

$$x^* = \bar{x}+\frac{b_1}{b_0+b_1}(\bar{y}-y)-\frac{k_M}{2(b_0+b_1)(w-c)}. \tag{10}$$

Solving (9) and (10) simultaneously results in the Nash solution. Then we can formulate a matrix form for the equilibrium:

$$\begin{bmatrix} 1 & 1 \\ 1 & b_1/(b_0+b_1) \end{bmatrix} \begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} \bar{x}+\bar{y}-k_R/2b_1(p+r-w) \\ \bar{x}+b_1\bar{y}/(b_0+b_1)-k_M/2(b_0+b_1)(w-c) \end{bmatrix}.$$

Let

$$\begin{bmatrix} 1 & 1 \\ 1 & b_1/(b_0+b_1) \end{bmatrix} = \mathrm{A}.$$

Note that $\det[A] = (b_1/b_0+b_1)-1 = (-b_0/b_0+b_1) < 0$. Thus, A has an inverse matrix with

$$A^{-1} = \frac{1}{b_0}\begin{bmatrix} -b_1 & (b_0+b_1) \\ (b_0+b_1) & -(b_0+b_1) \end{bmatrix}.$$

Thus

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \frac{1}{b_0}\begin{bmatrix} -b_1 & (b_0+b_1) \\ (b_0+b_1) & -(b_0+b_1) \end{bmatrix}\begin{bmatrix} \bar{x}+\bar{y}-k_R/2b_1(p+r-w) \\ \bar{x}+b_1\bar{y}/(b_0+b_1)-k_M/2(b_0+b_1)(w-c) \end{bmatrix}$$

$$= \begin{bmatrix} \bar{x}+k_R/2b_0(p+r-w)-k_M/2b_0(w-c) \\ \bar{y}-k_R(b_0+b_1)/2b_0b_1(p+r-w)+k_M/2b_0(w-c) \end{bmatrix}. \quad \square$$

A.2. Proof of Proposition 2

From Proposition 1, $x^* = \bar{x}$ and $y^* = \bar{y}$ require

$$\frac{k_R}{2b_0(p+r-w)} = \frac{k_M}{2b_0(w-c)} \tag{11}$$

and

$$\frac{k_R(b_0+b_1)}{2b_0b_1(p+r-w)} = \frac{k_M}{2b_0(w-c)}. \tag{12}$$

From (11) and (12), we obtain $(b_0+b_1/b_1)=1$. Then $b_0=0$.

Also, plugging $b_0=0$ into the demand functions (1) and (2) and then solving them simultaneously, we obtain $x^* = \bar{x}$ and $y^* = \bar{y}$.

(b) From Proposition 1 $x^*+y^* = \bar{x}+\bar{y}+\frac{k_R}{2b_0(p+r-w)}-\frac{k_M}{2b_0(w-c)}-\frac{k_R(b_0+b_1)}{2b_0b_1(p+r-w)}+\frac{k_M}{2b_0(w-c)} = \bar{x}+\bar{y}-\frac{k_R}{2b_1(p+r-w)} < \bar{x}+\bar{y}$. □

A.3. Proof of Proposition 3

The FOC of Eq. (3):$(\partial \Pi(x,y)/\partial y)(p+r-c)2b_1(\bar{x}+\bar{y}-x-y)-k_R$ $(-b \pm \sqrt{b^2-4ac}/2a) \underset{set}{=} 0$. Then

$$y = \bar{x}+\bar{y}-x-\frac{k_R}{2b_1(p+r-c)}, \tag{13}$$

$$\frac{\partial \Pi(x,y)}{\partial x} = (p-c)2b_0(\bar{x}-x)+(p+r-c)2b_1(\bar{x}+\bar{y}-x-y)-k_M =_{set} = 0.$$

Plug-in (13) into this FOC. Then

$$x = \bar{x}+\frac{k_R-k_M}{2b_0(p-c)}. \tag{14}$$

Then, plug-in (14) into (13):

$$y = -\frac{k_R-k_M}{2b_0(p-c)}-\frac{k_R}{2b_1(p+r-c)}. \quad \square$$

A.4. Proof of Proposition 4

(a) If $b_0=0$, Eq. (3) can be revised as $\Pi(x,y) = (p-c)a_0-k_Mx+(p+r-c)\{a_1+b_1(x+y)(2\bar{x}+2\bar{y}-x-y)\}-k_Ry$. The FOCs of revised profit will be

$$(p+r-c)2b_1(\bar{x}+\bar{y}-x-y)-k_M = 0 \tag{15}$$

for x and

$$(p+r-c)2b_1(\bar{x}+\bar{y}-x-y)-k_R = 0 \tag{16}$$

for y. In order to satisfy (15) and (16) under the condition $b_0=0$, $k_M=k_R$ is required.

(b) $x^0+y^0 = \bar{x}+\bar{y}-(k_R/2b_1(p+r-c)) < \bar{x}+\bar{y}$. From Proposition 1 $x^*+y^* = \bar{x}+(k_R/2b_0(p+r-w))-(k_M/2b_0(w-c))+\bar{y}-(k_R(b_0+b_1)/2b_0b_1(p+r-w))+(k_M/2b_0(w-c)) = \bar{x}+\bar{y}(k_R/2b_1(p+r-w))$. From assumption, $p+r-c > p+r-w$. Thus $(k_R/2b_1(p+r-w)) > (k_R/2b_1(p+r-c)) > 0$. Consequently $x^*+y^* < x^0+y^0$ and $x^0+y^0 < \bar{x}+\bar{y}$. □

A.5. Proof of Proposition 5

First, the retailer's response for given x is

$$y^* = y^*(x) = \bar{x}+\bar{y}-x-\frac{k_R}{2b_1(p+r-w)}. \tag{17}$$

Note that y^* represents a response. Plug (17) into the manufacturer's profit function

$$\pi_M(x|y^*(x))$$
$$= (w-c)\left\{a_0+b_0x(2\bar{x}-x)+a_1+b_1\left(x+\left(\bar{x}+\bar{y}-x-\frac{k_R}{2b_1(p+r-w)}\right)\right)\times\left(2\bar{x}+2\bar{y}-x-\left(\bar{x}+\bar{y}-x-\frac{k_R}{2b_1(p+r-w)}\right)\right)\right\}-k_Mx$$
$$= (w-c)\left\{a_0+b_0x(2\bar{x}-x)+a_1+b_1\left(\bar{x}+\bar{y}-\frac{k_R}{2b_1(p+r-w)}\right)\times\left(\bar{x}+\bar{y}+\frac{k_R}{2b_1(p+r-w)}\right)\right\}-k_Mx. \tag{18}$$

The FOC for (18) is

$$\frac{\partial \pi_M(x)}{\partial x} = 2b_0(w-c)(\bar{x}-x)-k_M \underset{set}{=} 0.$$

Then

$$x^{**} = \bar{x}-\frac{k_M}{2b_0(w-c)}. \tag{19}$$

Plug x^{**} of (19) into x of (17). The Stackelberg solution for a retailer is determined as

$$y^{**} = \bar{y}+\frac{k_M}{2b_0(w-c)}-\frac{k_R}{2b_1(p+r-w)}. \quad \square$$

A.6. Proof of Proposition 6

(a) We assume $b_0 > 0$, $k_M > 0$, and $w-c > 0$. If so, always $y^{**} < \bar{y}$. However, if $(k_M/2b_0(w-c)) = (k_R/2b_1(p+r-w))$, that is, the model structure is symmetric for a manufacturer and a retailer, then $y^{**} = \bar{y}$.

(b) Set $b_0=0$ and then solve the profit functions. First, there is no change for the retailer's FOC: $y^*(x) = \bar{x}+\bar{y}-x-(k_R/2b_1(p+r-w))$. Next, the manufacturer's FOC will be $\pi_M(x|y^*(x)) = -k_Mx < 0$. Consequently, we obtain $x^{**}=0$ and $y^{**} = \bar{x}+\bar{y}-(k_R/2b_1(p+r-w))$.

(c) From Proposition 3

$$x^{**}+y^{**} = \bar{x}-\frac{k_M}{2b_0(w-c)}+\bar{y}+\frac{k_M}{2b_0(w-c)}-\frac{k_R}{2b_1(p+r-w)}$$
$$= \bar{x}+\bar{y}-\frac{k_R}{2b_1(p+r-w)} < \bar{x}+\bar{y}. \quad \square$$

A.7. Proof of Proposition 7

(a) From (5) $(\partial \pi_M^+/\partial x) = (w-c)\{2b_0(\bar{x}-x)+2b_1(\bar{x}+\bar{y}-x-y+\gamma r)\}-k_M \underset{set}{=} 0$. Then

$$(b_0+b_1)x+b_1y = (b_0+b_1)\bar{x}+b_1\bar{y}-\frac{k_M}{2(w-c)}+b_1\gamma r. \tag{20}$$

From (6) $(\partial \pi_R^+/\partial y) = (p+r-w)2b_1(\bar{x}+\bar{y}-x-y+\gamma r)-k_R \underset{set}{=} 0$. Then

$$x+y = \bar{x}+\bar{y}-\frac{k_R}{2b_1(p+r-w)}+\gamma r. \tag{21}$$

From (20) and (21), the equilibrium levels, x^+ and y^+, are determined as

$$\begin{bmatrix} b_0+b_1 & b_1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x^+ \\ y^+ \end{bmatrix} = \begin{bmatrix} b_0+b_1 & b_1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \begin{bmatrix} -k_M/2(w-c) \\ -k_R/2b_1(p+r-w) \end{bmatrix} + \begin{bmatrix} b_1\gamma r \\ \gamma r \end{bmatrix}.$$

Finally

$$\begin{bmatrix} x^+ \\ y^+ \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \frac{1}{b_0}\begin{bmatrix} 1 & -b_1 \\ -1 & b_0+b_1 \end{bmatrix}\begin{bmatrix} -k_M/2(w-c) \\ -k_R/2b_1(p+r-w) \end{bmatrix} + \frac{1}{b_0}\begin{bmatrix} 1 & -b_1 \\ -1 & b_0+b_1 \end{bmatrix}\begin{bmatrix} b_1\gamma r \\ \gamma r \end{bmatrix}$$
$$= \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \begin{bmatrix} k_R/2b_0(p+r-w)-k_M/2b_0(w-c) \\ -k_R(b_0+b_1)/2b_0b_1(p+r-w)+k_M/2b_0(w-c) \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma r \end{bmatrix}. \quad (22)$$

(b) Comparing x^+ in Eq. (22) with x^* in Proposition 1, it is obvious that $x^+ = x^*$.

(c) Comparing y^+ in Eq. (22) with y^* in Proposition 1, the only difference is the last term in (22) (i.e., γr). We assume $r>0$. Hence, clearly, if $\gamma>0$, then $y^+>y^*$, and if $\gamma=0$, then $y^+=y^*$.

(d) Setting $b_0=0$, (21) will be modified to

$$\begin{bmatrix} b_1 & b_1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x^+ \\ y^+ \end{bmatrix} = \begin{bmatrix} b_1 & b_1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} + \begin{bmatrix} -k_M/2(w-c) \\ -k_R/2b_1(p+r-w) \end{bmatrix} + \begin{bmatrix} b_1\gamma r \\ \gamma r \end{bmatrix}. \quad (23)$$

To achieve $\begin{bmatrix} x^+ \\ y^+ \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}$ in (23), it is necessary to satisfy $\begin{bmatrix} k_M/2(w-c) \\ k_R/2b_1(p+r-w) \end{bmatrix} = \begin{bmatrix} b_1\gamma r \\ \gamma r \end{bmatrix}$. Equivalently, it is necessary to satisfy $(k_M/w-c) = (k_R/p+r-w)$. □

A.8. Proof of Proposition 8

(a) FOC for (8):

$$\frac{\partial \pi_R^I(y|x)}{\partial y} = (p+r-w)(1+\beta x)(2b_1) \times \{\bar{x}+\bar{y}+\beta\bar{x}\bar{y}-x-y-\beta xy\} - k_R \underset{set}{=} 0. \quad (24)$$

FOC for (7):

$$\frac{\partial \pi_M^I(y|x)}{\partial x} = (w-c)\{2b_0(\bar{x}-x)+2b_1(1+\beta y) \times (\bar{x}+\bar{y}+\beta\bar{x}\bar{y}-x-y-\beta xy)\} - k_M \underset{set}{=} 0. \quad (25)$$

From (24), we know

$$(1+\beta x)(2b_1)\{\bar{x}+\bar{y}+\beta\bar{x}\bar{y}-x-y-\beta xy\} = k_R/(p+r-w). \quad (26)$$

Thus, plugging (26) into (25), we obtain

$$\frac{k_R(1+\beta y)}{p+r-w} - k_M(1+\beta x) - 2b_1(w-c)(\bar{x}-x)(1+\beta x) = 0. \quad (27)$$

Eq. (27) is expressed as a quadratic function with respect to x in the x–y plane, but it is also a function of β, where $0<\beta<1$. From (27)

$k_R(1+\beta y)/(p+r-w) = k_M(1+\beta x) - 2b_1(w-c)(\bar{x}-x)(1+\beta x)$.

Hence, $1+\beta y = (1+\beta x)\left[k_M - 2b_1(w-c)(\bar{x}-x)\right]\left[\frac{p+r-w}{k_R}\right]$, and then

$$y = \left(\frac{1}{\beta}+x\right)\left[k_M-2b_1(w-c)(\bar{x}-x)\right]\left[\frac{p+r-w}{k_R}\right] - \frac{1}{\beta}.$$

Let $A(x) = \left[k_M-2b_1(w-c)(\bar{x}-x)\right]\left[\frac{p+r-w}{k_R}\right]$, then $y_\beta = \frac{\partial y}{\partial \beta} = \left(\frac{1}{\beta^2}\right)[1-A(x)]$. Hence, if $A(x)<1$, then y_β is an increasing function of β. Note that k_M is a marginal cost for after-sales service that the manufacturer bears and $2b_1(w-c)(\bar{x}-x)$ can be interpreted from (2) as a marginal production cost that the manufacturer bears. It is natural for durable goods and electronics that the marginal production cost is greater than marginal cost for after-sales service. Consequently, it is reasonable to assume that the sign of $A(x)$ is negative. Hence, $A(x)<1$ is satisfied in reasonable situations. In conclusion, y_β increases as β increases under the condition $A(x)<1$. Finally, equilibrium x^I and y^I increase in β.

(b) It is obvious that $\bar{x}$ and $\bar{y}$ do not satisfy the FOCs (23) and (24) when either $b_0=0$ or $b_0 \neq 0$. □

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